

Section 6.1: Composition of Functions

Objectives:

- 6.1: Evaluate composite functions.
- 6.1: Form composite functions and find their domains.
- 6.1: Find functions that form given composite functions.
- 6.1: Solve applications involving composite functions.

Definition: Composition Function

Given two functions f and g , the **composite function**, denoted by $f \circ g$ (read as “ f composed with g ”), is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f .

Ex $f(x) = x + 2$; $g(x) = x^2 + 2x - 6$

Find: a) $(f \circ g)(1) = f(g(1))$
 $= f(-3)$
 $= -1$

b) $(g \circ f)(2) = g(f(2))$
 $= g(4) = 18$

Ex

x	0	1	2	3
f(x)	10	12	15	20
g(x)	5	2	4	6

$$\begin{aligned}(f \circ g)(1) &= f(g(1)) \\ &= f(2) \\ &= 15\end{aligned}$$

Example 2: Evaluate Composition Functions

For the given functions f and g , evaluate the following:

$$f(x) = 6\sqrt{x}; g(x) = 2x$$

$$\begin{aligned} \text{(a)} \quad (f \circ g)(4) &= \frac{f(g(4))}{g(f(2))} = f(8) = 6\sqrt{8} = 12\sqrt{2} \\ \text{(b)} \quad (g \circ f)(2) &= \frac{g(f(2))}{f(f(1))} = g(6\sqrt{2}) = 12\sqrt{2} \\ \text{(c)} \quad (f \circ f)(1) &= \frac{f(f(1))}{g(g(0))} = f(6) = 6\sqrt{6} \\ \text{(d)} \quad (g \circ g)(0) &= \frac{g(g(0))}{f(f(1))} = g(0) = 0 \end{aligned}$$

Your Turn

For the given functions f and g , evaluate the following:

$$f(x) = x^2 + 10; g(x) = \sqrt{x - 1}$$

$$\begin{aligned} (a) (f \circ g)(5) &= \underline{f(g(5))} = f(2) = 14 \\ (b) (g \circ f)(4) &= \underline{g(f(4))} = g(26) = 5 \\ (c) (f \circ f)(0) &= \underline{f(f(0))} = f(10) = 110 \\ (d) (g \circ g)(2) &= \underline{g(g(2))} = g(1) = 0 \end{aligned}$$

Composite Functions and their Domains

Let $f(x) = (x - 3)^2$ and $g(x) = \frac{1}{2x}$.

a) Find $g(f(x)) = \frac{1}{2(x-3)^2}$

b) Find the domain of the composite function $g(f(x))$. $D: x \neq 3$

Example 3: Form Composite Functions and Find Their Domains

Given the functions $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x}$, determine $f \circ g$ and state its domain.

$$f \circ g = f(g(x)) = \frac{1}{\left(\frac{1}{x} - 1\right)} \cdot \frac{x}{x} = \frac{x}{1-x}$$

Answer: $f \circ g = \frac{x}{1-x}$

Domain in set notation: $\{x \mid x \neq 1, 0\}$

Your Turn

Given the functions $f(x) = \frac{7}{x-6}$ and $g(x) = \frac{9}{x+1}$, determine $f \circ g$ and state its domain.

$$f \circ g = \frac{7}{\left(\frac{9}{x+1} - 6\right)} \cdot \frac{(x+1)}{(x+1)} = \frac{7x+7}{9-6(x+1)} = \frac{7x+7}{9-6x-6}$$

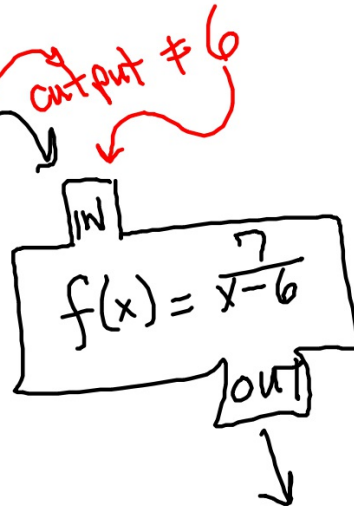
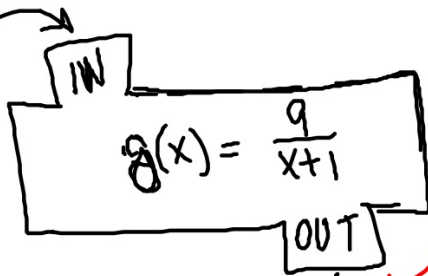
$$\text{Answer: } f \circ g = \frac{7x+7}{-6x+3}$$

$$\begin{aligned} -6x+3 &= 0 \\ x &= \frac{1}{2} \end{aligned}$$

$$\text{Domain in set notation: } \{x \mid x \neq \frac{1}{2}, -1\}$$

$$f \circ g = f(g(x))$$

$$\begin{aligned} x &\neq -1 \\ x &\neq \frac{1}{2} \end{aligned}$$



$$\frac{9}{x+1} \neq 6$$

$$9 \neq 6(x+1)$$

$$9 \neq 6x+6$$

$$3 \neq 6x$$

$$\frac{1}{2} \neq x$$

output $\neq 6$

Example 4: Form Composite Functions and Find Their Domains

Given the functions $f(x) = \sqrt{x - 2}$ and $g(x) = x^2 + 7$, determine $f \circ g$ and state the domain of $f \circ g$.

$$f \circ g = \sqrt{x^2 + 7 - 2} = \sqrt{x^2 + 5}$$

$$D: \text{all Reals}$$

$$D: \begin{array}{l} x^2 + 5 \geq 0 \\ x^2 \geq -5 \\ x \geq \pm\sqrt{5} \end{array}$$

Example 5: Solve Applications

If the radius of the oil spill (in miles) is given by

$$r(t) = 10t - 0.2t^2$$

where t is in days, and the area of the oil spill is given by

$$A(r) = \pi r^2$$

(a) Find a function that gives the area of the oil spill in terms of the number of days since the start of the spill, $A(r(t))$ or simply $A(t)$. $= \pi(10t - 0.2t^2)^2$

(b) Find the area of the oil spill to the nearest square mile 7 days after the start of the spill. $A(7) \approx 11,385 \text{ mi}^2$